Complexity of Bi-Objective Buffer Allocation Problem in Systems with Simple Structure

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Abstract. We consider a bi-objective optimization problem of choosing the buffers capacity in a production system of parallel tandem lines, each consisting of two machines with a single intermediate buffer. During operation of the system, the equipment stops occur due to failures and these stops are random in the moments when they arise and in their durations. The product is accumulated in an intermediate buffer if the downstream machine is less productive than the upstream machine.
We study the complexity of exact and approximate computations of a Pareto front for the following two bi-objective problem formulations: (i) the expected revenue maximization with minimization of buffers allocation cost and (ii) the expected revenue maximization with minimization of expected inventory costs. The expected revenue is assumed to be an increasing function of the expected throughput of the system.
On the one hand, fully polynomial-time approximation schemes for approximation of Pareto fronts of these problems are proposed and an exact pseudo-polynomial time algorithm is suggested for the first problem in the case of integer buffer capacity costs. On the other hand, we show that both of these problems are intractable even in the case of just one tandem two-machine line.

Keywords: inventory system, throughput, capital costs, storage costs, intractability

1 Introduction

Finding the set of Pareto-optimal solutions or a close approximation to it are of great importance in design of automated control and decision support systems. The problem of buffer volume optimization of the volume of buffers arises in management of such manufacturing systems as automatic lines, flexible production systems and automated assembly lines, where parts are moved from one machine to another using some transport mechanism.

Due to equipment failures, in the process of operation of the line the machine breakdowns occur in random moments and have a random duration. The consequences of failures spread on related operations due to the impossibility to pass an item onto the following operation, or lack of parts coming from the upstream machine. Presence of buffers for storage of parts between the machines allows to reduce the impact of failures on neighboring operations, and to increase the line throughput, i.e. the production rate of the line in the stationary regime. However, installation of buffers is associated with additional capital expenditures and increases the inventory of parts. The problem consists in choosing the volume of buffers based on the throughput of the line, the capital cost of the installation of buffers and the inventory cost.

Significance of solving such problems of optimization of production lines is shown in [28]. Economic effect of implementation of solution methods for such problems in the car production is shown in [26] on the example of PSA Peugeot Citroën.

Analysis of production lines subject to failures is usually conducted using Markov models with discrete or continuous time under the assumption of geometric or exponential distributions of time to fail and time to repair (see [10]). The duration of processing a part can be assumed deterministic or random (typically with geometric, Erlang or the exponential probability distribution). In the case of continuous time and deterministic durations of parts processing, some non-Markov transitions may be approximated by Markov transitions under the assumption of exponential distribution of the corresponding random variables [16, 23, 17]. At quite natural assumptions thus obtained Markov models have a stationary distribution (see [24], Chapter 2) and the throughput as well as the expected number of parts in each buffer are determined in the stationary regime.

Most of the works in the literature on optimization of buffer volumes are dealing with a single-criterion problem formulations (see [2, 18, 20]). Other studies consider more than one criterion, but using a weighted sum of criteria [1, 12]. In [5], the ant colony algorithm and the evolutionary algorithm of [30] are adapted for multi-criteria buffer allocation problem. Here the optimization criteria are maximization of the throughput of the line, calculated with a simulation algorithm, and minimization of the overall buffers volume. The well-known variant of multi-objective genetic algorithm [11] is adapted in [9] to the bi-criteria buffers allocation problem, where the criteria are the throughput and the capital cost of buffers installation.

In the present paper, we consider three criteria: maximization of average production rate in the steady regime, minimization of capital costs for the installation of buffers and minimization of the average inventory cost for storage of parts in the intermediate buffers.

Exact methods of calculation of the average production rate are known for two-machine tandem lines and, in some special cases, for the three three-machine tandem lines (see e.g., the review [10]). For the general case, one can only apply the approximate decomposition methods,approximate aggregation or simulation methods [10, 16].

In the present paper, we will not assume a specific type of distribution of time to fail and time to repair or processing time of machines. Neither shall we choose a specific method of computing the expected throughput and inventory of a line. Instead of that, we will make two simple monotonicity assumptions which hold in many different versions of the buffers allocation problem (see the details below in Section 2).

Suppose that on the set of feasible solutions D, the vector function of criteria $\mathbf{f} = (f_1, f_2)$ is specified with points $\mathbf{f}(x) = (f_1(x), f_2(x)) \in \mathbb{R}^2, x \in D$ in the criteria values space. In our case, f_1 is a maximization criterion and f_2 is a minimization criterion. Let us define the Pareto dominance in the space $\mathbb{R}^2$: vector $\mathbf{f} = \mathbf{f}(x)$, $x \in D$ is Pareto-dominated by vector $\bar{\mathbf{f}} = \mathbf{f}(\bar{x})$, $\bar{x} \in D$, if the inequalities $f_1(x) \geq f_1(\bar{x})$, $f_2(x) \leq f_2(\bar{x})$ hold and there is at least one strict inequality among them. A solution $x \in D$ is dominated by a solution $\bar{x} \in D$, if the vector $\mathbf{f}(x)$ is dominated by vector $\mathbf{f}(\bar{x})$ in the sense of Pareto. The set $\tilde{D}$ of all non-dominated feasible solutions is called the *set of Pareto-optimal solutions.* The *total set of alternatives* is a subset $D^0 \subseteq \tilde{D}$ of a minimum size, such that $f(D^0) = f(\tilde{D})$ [27]. The *Pareto Front* is the set $F := \mathbf{f}(\tilde{D})$. Given $\varepsilon > 0$, the *Pareto set ε-approximation* $\tilde{D}_\varepsilon$ is a set such that for any Pareto optimal solution $\tilde{x} \in \tilde{D}$, there is a solution $x \in \tilde{D}_\varepsilon$ satisfying $f_1(x) \geq (1-\varepsilon)f_1(\tilde{x})$ and $f_2(x) \leq (1+\varepsilon)f_2(\tilde{x})$.

In what follows, m denotes the number of machines in a production line, N is the number of intermediate buffers, subject to optimization.

By *system with a simple structure*, we mean a system which consists of N parallel two-machine tandem lines with common input buffer and common output buffer. An example of a system with simple structure is provided in Figure 1.

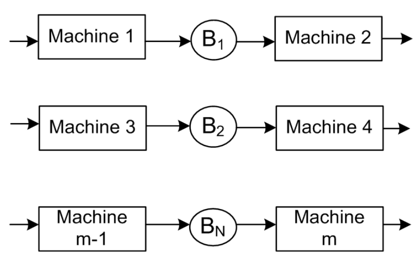

Fig. 1. Example of a series-parallel line with simple structure (N two-machine tandem lines in parallel)

In what follows, we consider the complexity of two bi-objective optimization problems that ask to determine the buffers capacity in a production system with simple structure. The optimization criteria we consider are the same as in [13]: the expected revenue due to line operation, the capital costs for installing buffers, and the expected total inventory cost for intermediate products. The expected revenue is supposed to be an increasing function of the expected throughput of the system.

On the positive side, we propose two fully polynomial-time approximation schemes (FPTASes) for approximation of Pareto front in the following two bi-objective problem formulations: (i) the expected revenue maximization with minimization of capital costs and (ii) the expected revenue maximization with minimization of the expected inventory costs. An exact pseudo-polynomial time algorithm is proposed for computing the Pareto front in the first problem, if the buffers allocation cost is a linear function of buffer sizes with integer coefficients, i.e. assuming integer buffer capacity costs.

On the negative side, we show that the canonical decision problems for the above mentioned bi-objective problems are $\mathcal{NP}$-hard even if the revenue is proportional to the production rate and the buffers allocation cost is linear. In the case of just one tandem two-machine line, both of the problems are complete multiobjective optimization problems in the sense of Emelichev and Perepelitsa [19] and therefore intractable, i.e. their Pareto-front can be of exponential size in the input size. We also show for both of these special cases that if the Pareto front is computable in a polynomial time then $\mathcal{P} = \mathcal{NP}$ holds.

The remainder of the paper is organized as follows. The assumptions of the model of production line and the bi-objective problems formulation are presented in the next section. Section 3 is devoted to the analysis of computational complexity of bi-objective buffer allocation problems on the two-machine tandem lines. This is followed by the analysis of computational complexity and approximability of bi-objective buffer allocation problems for lines of simple structure in Section 4. Finally some conclusions are drawn in Section 5.

2 Basic Properties and Definitions

2.1 An Illustrative Model of Production Line

Let us consider an illustrative example of a production systems under consideration. Suppose that each machine of the system can be in an operational state or under repair. An operational machine may be blocked and temporarily stopped in case if there is no room in the downstream buffer. An operational machine may be starved if there are no parts to process in the upstream buffer. Otherwise operational machines are working.

A working machine is assumed to have a constant cycle time. It is supposed that machines break down only when they are working. The times to fail and times to repair for each machine are assumed to be mutually independent and exponentially distributed random values. A detailed analysis of steady-state performance of such systems and optimization of its parameters were carried out in a number of works, see e.g. [3, 8, 12–17, 23].

2.2 Optimization Criteria and the Set of Feasible Solutions

Let the buffers in the system be denoted by $B_1, \dots, B_N$ and let h_j be the capacity of buffer B_j, $j = 1, ..., N$, subject to optimization. Denote the vector of decision variables by $H = (h_1, h_2, \dots, h_N) \in \mathbb{Z}_+^N$, where $\mathbb{Z}_+$ is the set of non-negative integers. Let $D = \{H = (h_1, ..., h_N) \in \mathbb{Z}_+^N \mid 0 \le h_i \le d_i, i = 1, ..., N\}$ be the set of feasible solutions, where $d_1, \dots, d_N$ are the maximal admissible buffer capacities.

The most commonly used optimization criteria are:

- the throughput, i.e. expected number of parts produced by the system per unit of time in the steady state mode (expected steady state production rate) $V(H)$;
- the expected steady state inventory $Q(H) = (q_1(H), \dots, q_N(H))$, where $q_j(H) \in [0, h_j]$ is the expected steady state number of parts in buffer B_j, $j = 1, \dots, N$.

Let us introduce the following additional notation, using the symbol $\mathbb{Q}$ for the set of rational numbers:

- $R(V)$ is the revenue related to the production rate V, i.e. $R : \mathbb{Q}_+ \to \mathbb{Q}_+$;
- $B(H)$ is the cost of buffer configuration H, i.e. $B : D \to \mathbb{Q}_+$;
- $C(Q)$ is the cost of expected steady state inventory vector Q, i.e. $C : \mathbb{Q}_+^N \to \mathbb{Q}_+$.

In what follows, $R(V)$ is assumed to be a given non-decreasing function. In the case of lines with simple structure, $V(H)$, $B(H)$ and $C(Q)$ are assumed to be given completely additively separable functions, non-decreasing in each argument. Recall that $f(x_1, \dots, x_n)$ is called completely additively separable if $f(x) = f(x_1) + \dots + f_n(x_n)$ for some functions $f_1, \dots, f_n$, each a function of one variable. We also make two technical assumptions: (i) functions $V(H)$, $B(H)$, $C(Q)$ $Q(H)$ and $R(V)$ are computable in polynomial time, and (ii) denoting any of these functions by $f(\cdot)$, we have the value $|\log f(\cdot)|$ polynomially bounded in the length of the problem input.

The cost function $B(H)$ may be non-linear to model some standard buffer capacities. A stepwise revenue function can be used to model zero revenue in case of an unacceptably low throughput.

2.3 Formulation of the Bi-Objective Problems

Let us use the following notation for the problems of finding a Pareto front:

- In (R,B)-Pareto, the criterion f_1 is the expected revenue maximization $R(V(H))$ and f_2 is the minimization of buffer allocation cost $B(H)$.
- In (R,C)-Pareto, the criterion f_1 is the expected revenue maximization $R(V(H))$ and f_2 is the minimization of expected inventory cost $C(Q(H))$.

In accordance with [4, 25, 29], by *Canonical Decision Problem* for the buffers allocation problem with criteria pairs $f_1(H) \to \max$, $f_2(H) \to \min$, we mean the following decision problem: Given an instance I of buffer allocation problem and a pair $(\alpha, \beta) \in \mathbb{R}_+^2$, decide whether there exists a feasible buffers allocation H', for which $f_1(H') \ge \alpha$, $f_2(H') \le \beta$.

Let us use the following notation for the canonical decision:

– In (V,linear B)-Dec, the criterion $f_1(H) \equiv V(H)$ and f_2 is the minimization of linear buffers allocation cost $B(H) = \sum_{i=1}^{N} b_i h_i$.
– In (V,linear Q)-Dec the criterion $f_1(H) \equiv V(H)$ and f_2 is the minimization of linear inventory cost $C(Q(H)) = \sum_{i=1}^{N} c_i q_i(H)$.

2.4 Monotonicity Properties

In what follows, we use the following two monotonicity assumptions for each subsystem consisting of two machines, separated by a buffer B_i of size h_i:

– **M1. Monotonicity of expected throughput.** $V(H)$ is an increasing function of $h_i,\ i = 1, \dots, N$.
– **M2. Monotonicity of expected inventory.** $q_i(H)$ is an increasing function of $h_i,\ i = 1, \dots, N$.

In the case of illustrative model presented in Subsection 2.1, properties M1 and M2 follow from the analytical solution of the system of Kolmogorov equations describing the two-machine production system [8, 17].

3 Computational Complexity of Bi-Objective Buffer Allocation Problems on Two-Machine Tandem Line

In the case of $N = 1$, we denote $h := h_1 = H$ and $d := d_1$ for simplicity. Let us consider two increasing functions $V(h)$ and $Q(h)$, defined for $h = 0, 1, \dots, d$, and taking rational values.

Theorem 1. *(i) If problem* (R,B)-Pareto *is polynomially solvable in case of* $N = 1$*, then* $\mathcal{P} = \mathcal{NP}$.
(ii) If problem (R,C)-Pareto *is polynomially solvable in case of* $N = 1$*, then* $\mathcal{P} = \mathcal{NP}$.

The proof is similar to the proof of Proposition 1 in [13] and employs an idea of Cheng and Kovalyov [7].

In [19], Emelichev and Perepelitsa give a definition of the *complete multiobjective problem*. A multiobjective optimization problem with k objectives is called *complete* if for any instance I of this problem with a set of feasible solutions D, there exists a vector of criteria $(f_1, \dots, f_k)$, such that D is the only total set of alternatives w.r.t. $(f_1, \dots, f_k)$, i.e. $D^0 = D$ holds.

In view of the monotonicity assumptions for $V(h)$ and $Q(h)$, a straightforward verification of the above definition indicates that (R,B)-Pareto and (R,Q)-Pareto are both complete in the sense of Emelichev and Perepelitsa. Note that d is a numerical parameter of the problem and $|D| = d + 1$. Together with the completeness property, this implies that the cardinalities of Pareto fronts of (R,B)-Pareto and (R,Q)-Pareto are not bounded by any polynomial in problem input size and therefore these problems are *intractable*, according to the terminology from [4, 21].

4 Computational Complexity of Bi-Objective Buffer Allocation Problems on Lines of Simple Structure

Consider the bi-objective buffer allocation problem with criteria of expected revenue $R(V(H))$ maximization and buffers installation cost $B(H)$ minimization on the set D, assuming that $B(H) = \sum_{i=1}^{N} b_i h_i$ where $b_i \in \mathbb{Z}$ for all $i = 1, \ldots, N$.

Proposition 1. *In the case of lines of simple structure, the problem* (V,linear B)-Dec *is $\mathcal{NP}$-hard.*

The proof of Proposition 1 is analogous to the proof [15] of $\mathcal{NP}$-hardness of the single-objective buffer allocation problem with the criterion of $B(H)$ minimization, given a lower bound on $V(H)$.

Proposition 2. *In the case of lines of simple structure, the problem* (V,linear Q)-Dec *is $\mathcal{NP}$-hard.*

The proof of Proposition 2 is analogous to the proof [13] of $\mathcal{NP}$-hardness of the single-objective buffer allocation problem with the criterion of $Q(H)$ minimization, given a lower bound on $V(H)$.

Proposition 3. *In the case of lines of simple structure, the Pareto front of buffer allocation problem with the criteria of expected revenue $R(V(H))$ maximization and buffers installation cost $B(H)$ minimization is computable in pseudopolymonial time, assuming integer b_i, $i = 1, \ldots, N$.*

The proof of Proposition 3 is based on the dynamic programming method. It is similar to the proof of pseudo-polynomial solvability of the Integer Knapsack Problem.

Analogous claim for an arbitrary function $B(H)$, increasing in each of its arguments h_i, is problematic. The reason is that the dynamic programming method gives pseudopolymonial solvability only if the cardinality of the set of values, taken by one of the optimization criteria, is polynomially bounded, given polynomially bounded numerical input data. The same difficulty arises in computing the Pareto set of the bi-objective buffer allocation problem with criteria of the expected revenue $R(V(H))$ maximization and the expected inventory cost $C(Q(H))$ minimization.

It is possible, however, to convert the FPTAS for Generalized Knapsack problem [22] into FPTASes for the bi-criteria optimization problems, where expected revenue $R(V(H))$ maximization is combined with buffers installation cost $B(H)$ minimization or with the expected inventory cost $C(Q(H))$ minimization. Recall that a family of algorithms $\{A_\varepsilon\}$ is called a fully polynomial approximation scheme (FPTAS) for a multiobjective optimization problem if for any input instance and any $\varepsilon > 0$, the algorithm A_ε runs in polynomial time w. r. t. the size of the input and $1/\varepsilon$, and outputs a Pareto set ε-approximation.

Theorem 2. *In the case of lines of simple structure, there exist FPTASes for* (R,B)-Pareto *and* (R,C)-Pareto.

The proof is based on the general scheme suggested by Cheng et al. in [6] for the construction of Pareto set ε-approximation of a bi-criteria problem.

5 Conclusions

We have established intractability of several special cases of buffer allocation problem in bi-objective formulation using the proof ideas developed for the analysis of single-criteria formulations of the problem. Our results apply to different particular models of production line, provided that two monotonicity conditions, formulated here, are satisfied. On the positive side, we propose fully polynomial-time approximation schemes for approximation of the Pareto front for two versions of buffers allocation problem and an exact pseudo-polynomial algorithm based on the dynamic programming method.

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